$$\frac{L(t)}{2\pi\rho_\infty cVv_0 e^{ivt}} = \frac{2^{-n}n!}{[(n/2)!]^2}\left[C(\omega) + \frac{i\omega}{n+2}\right] \quad n \text{ even} \tag{15a}$$

$$= \frac{2^{-n}n!C(\omega)}{[(n-1)/2]![(n+1)/2]!} \quad n \text{ odd} \tag{15b}$$

$$\frac{M(t)}{\pi\rho_\infty c^2Vv_0 e^{ivt}} = \frac{2^{-n}n!}{[(n/2)!]^2}\left[C(\omega) - \frac{n}{n+2}\right] \quad n \text{ even} \tag{16a}$$

$$= \frac{2^{-n}n!}{[(n-1)/2]![(n+1)/2]!}\left[C(\omega) - 1 - \frac{i\omega}{n+3}\right] n \text{ odd} \tag{16b}$$

These agree with the classical results for $n = 0$, 1, and give very simple expressions for $n > 1$.

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Technical Comments

Comment on "Stability of Rotating Stratified Fluids"

Ulrich H. Kurzweg*
University of Florida, Gainesville, Fla.

IN a recent note Johnston[1] reported a new type of hydrodynamic instability observed in rotating flows with radially dependent density. He found that when a light gas (helium) is injected tangentially into a stationary gas of higher density (nitrogen) contained within a cylindrical test vessel that the resultant flow, as observed with aid of a laser-illuminated shadowgraph system, is unstable. The early stages of the instability are characterized by the appearance of nonaxisymmetric modes consisting of "two vortices which travel as well as rotate in the same direction as the main flow." The instability occurs under conditions where the product of the flow density and the square of its circulation is an increasing function of radius and hence, under conditions for which the classical Rayleigh–Synge criterion predicts stability for axisymmetric modes. The instability is observed to occur only when the density decreases with increasing radius in at least part of the flow region. No instabilities are found when a heavy gas is injected tangentially at the periphery of a gas core of lower density.

In attempting to explain these observations, Johnston[1] applied a modification of the Rayleigh–Synge criterion first proposed by Yih[2] and valid for flows where viscosity and thermal diffusivity play an important role in the instability growth mechanism. We wish to point out that such an application of Yih's criterion is incorrect since the criterion is applicable only to axisymmetric disturbances, while the instabilities observed are nonaxisymmetric as clearly shown in Fig. 2 of Johnston's paper. In addition, it is difficult to see how the dissipative effects of viscosity and thermal diffusivity can play a major role in the rapid disintegration process characterizing this instability phenomenon.

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* Associate Professor, Department of Engineering Science, Mechanics, and Aerospace Engineering.

A correct interpretation of the observed instabilities may be obtained by use of a general stability criterion given by us in an earlier note.[3] This criterion represents a sufficiency condition for the stability of heterogeneous swirling flows against arbitrary infinitesimal disturbances of axial wave number k and azimuthal wave number m and is valid in the absence of viscous and gravitational effects. For the rotating flows examined by Johnston, the sufficiency condition for flow stability given in Ref. 3 reduces to the simplified form

$$(\rho r^3)^{-1}(d/dr)[\rho(r^2\Omega)^2] + (m/rk)^2[r\Omega^2(d/dr)(\ln\rho) - \tfrac{1}{4}r^2(d\Omega/dr)^2] \geq 0 \tag{1}$$

where Ω and ρ represent the radially dependent angular velocity and density, respectively. For axisymmetric disturbances ($m = 0$) the inequality is identical with the Rayleigh-Synge criterion while for nonaxisymmetric modes with $k = 0$, the flow will generally be stable only when the radial density gradient is positive and sufficiently large to overcome the destabilizing effect of the angular velocity gradient. The nonaxisymmetric instabilities observed in Johnston's experiments occur for negative radial density gradient and under conditions where the Rayleigh–Synge criterion is satisfied. One sees that these conditions correspond precisely to the case for which inequality (1) is violated for nonaxisymmetric modes only. Although small violations of criterion (1) need not necessarily imply instability, negative radial density gradients can generally be expected to lead to nonaxisymmetric modes of instability and will do so without the need to invoke the dissipative effects of viscosity. Finally, it is interesting to note from criterion (1) that heterogeneous rotating flows can be guaranteed stable for all infinitesimal disturbances when the radial density gradient is positive and the angular velocity gradient remains small. These are precisely the conditions encountered in centrifuges.

References

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